tures to Study Adsorption on Chromia-Promoted Magnetite at Water-Gas Shift Temperatures," accepted for publication in *J. Catal.* (1981).
Kul'kova, N. V., and M. I. Temkin, "Kinetics of the Reaction of Conversion of Carbon Monoxide by Water Vapor," *Z. Fiz. Khim.*, **23**, 695 (1949).
Oki, S., J. Happel, M. Hnatow, and Y. Kaneko, "The Mechanism of the Water-Gas Shift Reaction over Iron Oxide Catalyst," Proc. Intern. Congr. Catal., No. 5, 173 (1973).
———, and R. Mezaki, "Identification of Rate-Controlling Steps for the Water-Gas Shift Reaction Over an Iron Oxide Catalyst," *J. Phys. Chem.*, **77**, 447 (1973).
Podolski, W. F., and Y. G. Kim, "Modeling the Water-Gas Shift Reaction," *Ind. Eng. Chem. Process Des. Develop.*, **13**, 415 (1974).
Rachkovskii, E. E., and G. K. Boreskov, "The Mechanism of Isotope Exchange in the System $H_2 - H_2O$ on Some Oxide Catalysts," *Kinet. Katal.*, **11**, 1410 (1970).
Riecke, V. E., and K. Bohnenkamp, "On the Kinetics of Oxidation and Reduction of Wustite within its Range of Existence," *Arch. Eisenhutten*, **40**, 717 (1969).
Shchibrya, G. G., N. M. Morozov, and M. I. Temkin, "The Kinetics and Mechanism of the Catalytic Reaction Between Carbon Monoxide and Steam," *Kinet. Katal*, **6**, 1057 (1965).
Stotz, V. S., "Investigations Concerning the Mechanism of the Water-Gas Shift Reaction on Wustite," *Ber. Bonsenges. Phys. Chem.*, **70**, 769 (1966).
Temkin, M. I., "Relaxation Rate of a Two-Stage Catalytic Reaction," *Kinet. Katal.*, **17**, 1095 (1976).
Wagner, E., "Adsorbed Atomic Species as Intermediates in Heterogeneous Catalysis," *Adv. in Catal.*, **21**, 323 (1970).

Manuscript received April 24, 1981; revision received September 16, and accepted October 13, 1981.

Complex Transitional Flows in Concentric Annuli

R. W. HANKS and J. M. PETERSON

Department of Chemical Engineering
Brigham Young University
Provo, UT 84602

Detailed measurements of radial variations of turbulence intensity were made with a hot film anemometer in a concentric annulus of aspect ratio $R_i/R_o = 0.0416$ for a Reynolds number range $1{,}200 \leq Re \leq 3{,}000$ using a Newtonian polyglycol water solution. Measurements were also made of excess entrance pressure gradients and the axial variation of pressure gradients.

Based on the results of these measurements, it is concluded that the proposition of Lea and Tadros (1931), Rothfus (1948) and others that a transitional flow regime develops with a zone of turbulence near the inner core surrounded by a zone of laminar flow near the outer wall is false. The two-critical Reynolds number transitional regime proposed by Hanks and Bonner (1971) is verified to exist but their mathematical model for the interpretation of its cause is disproved. It is shown that at the lower critical Reynolds number predicted by Hanks and Bonner (1971) and by Hanks (1980) a transition does occur. It is conjectured that the new flow may be a type of complex laminar bifurcation flow described by Joseph (1976), although the present data do not permit a definite conclusion to be reached. This complex laminar flow in turn undergoes a transition to turbulent flow at a second critical Reynolds number, higher than the first, in qualitative accord with Hanks and Bonner's (1971) and Hanks' (1980) predictions, although the latter are not quantitatively correct because they are based on inadequate models of the transitional flow field.

SCOPE

The concentric annulus flow geometry is one which has found considerable practical application in the process industries. The concentric annulus also presents a flow system which is still amenable to analysis. In this seemingly simple flow field some rather strange and puzzling phenomena occur. The most interesting of these are associated with the transition from laminar to nonlaminar flow.

Hanks and Bonner (1971) presented extensive experimental data for a number of annuli which showed that the transition from laminar to turbulent flow in concentric annuli occurred over a rather extended range of Reynolds numbers, and that two very distinct critical values of the Reynolds number can be observed. A similar phenomenon had been suspected much earlier by Lea and Tadros (1931) and by Rothfus (1948) and Walker et al. (1957), but the existence of the multiple critical Reynolds numbers had not been clearly demonstrated. The existence of two critical Reynolds numbers had also been predicted theoretically (Hanks and Bonner, 1971; Joseph, 1976; Hanks, 1980).

Rothfus and his coworkers (Rothfus, 1948; Rothfus et al., 1950, 1958; Croop and Rothfus, 1962) observed definite distortions in the shapes of velocity profiles in transitional flows in annuli. In particular they observed that the point of maximum velocity deviated from its theoretical laminar position in a certain definite fashion. This behavior was explained by those authors as being caused by the onset of turbulence in a macroscopic flow region surrounding the core while the flow region lying outside

J. M. Peterson is presently with Signetics Corp., Orem, UT.
0001-1541-82-6422-0800-$2.00.

the point of maximum velocity remained in laminar flow. This conceptual model was due originally to Lea and Tadros (1931).

Hanks and Bonner (1971) presented data on the shifting of the location of maximum velocity which contradicted those of Croop and Rothfus (1962) and Walker (1957). They also developed a theoretical model, based on the Lea-Tadros (1931)-Rothfus (1948) simultaneous turbulent core zone-laminar outer zone concept. This model predicted shifts in the location of maximum velocity which were in qualitative agreement with their data and contradictory of those of Croop (1958) and Walker (1957). The theory also predicted (Hanks and Bonner, 1971) friction factor curves which were in qualitative but not quantitative agreement with measured values.

Narvaez (1977) and Mildenhall (1979) examined the linear hydrodynamic stability of annular flows, confirming the conjectures of Mott and Joseph (1968) that multiple modes of instability occur in annuli. They found, however, no difference between the inner and outer zones of flow (inner being near the core, outer lying outside the location of maximum velocity) so far as linear hydrodynamic stability is concerned.

Despite all the studies of the stability and transitional flow behavior of Newtonian flow in concentric annuli, considerable confusion and controversy still exists. The most fundamental question which must yet be answered is whether or not the Lea-Tadros-Rothfus-Hanks-Bonner model of transitional flow consisting of a coexisting turbulent core region surrounded by a laminar region, both of macroscopic size, is valid. A second question of nearly equal importance is whether the location of maximum velocity in an annulus shifts during transitional flow and if so, how. The purpose of the present experimental investigation was to obtain a definitive answer to the first of these questions. The second is outside the scope of this study and is addressed separately elsewhere (Peterson and Hanks, 1981). Only after these fundamental questions are answered can further progress towards understanding the complex flow phenomena which occur in this deceptively simple flow channel be made.

CONCLUSIONS AND SIGNIFICANCE

An extensive series of experiments was performed in a concentric annulus designed to duplicate as nearly as possible the conditions of one of Hanks and Bonner's (1971) annuli. Pressure drop-flow rate measurements were made to confirm or refute the existence of the increased frictional resistance in transitional flow reported by Hanks and Bonner (1971). Turbulence intensities were measured with a hot film anemometer as functions of both radial position and flow rate in the annulus. As a result of these measurements a number of definite conclusions has been reached.

The existence of a transitional flow range in which increased frictional resistance occurs, as evidenced by a statistically significant increase in the product $f{\cdot}\mathrm{Re}$ (Hanks and Bonner, 1971), was verified. This increase is real, stable, and reproducible. Over about 80% of the same Reynolds number range where the mild increase (3.2% for this particular annulus) in $f{\cdot}\mathrm{Re}$ occurs there is no measurable turbulence *anywhere* in the flow field. Thus, the model of a turbulent core zone surrounded by a laminar outer zone is incorrect. Transition to turbulence, when it finally happens, apparently occurs by the usual mechanism of corruption of the flow by intermittent bursts of turbulence (Rotta, 1956; Wygnanski and Champagne, 1973) whose frequency of occurence increases with increasing flow rate. The flow which occurs in the special transition range in annuli is a complex laminar flow, possibly of the bifurcation type discussed by Joseph (1976). Its origin and source of energy appears to be the entry of the annulus. Pressure gradients are axially invarient suggesting a fully developed condition. The exact details of this complex flow field are not known. Further, more sophisticated investigation will be required to determine these details.

INTRODUCTION

An increase in frictional pressure losses over that associated with rectilinear laminar flow but prior to a general transition to turbulence has previously been noted in concentric annuli by several investigators (Lea and Tadros, 1931; Rothfus, 1948; Walker et al., 1957; Hanks and Bonner, 1971). The characteristics of this increased pressure loss and the existence of two distinct critical Reynolds numbers defining this special transitional flow range are best displayed in terms of a plot of $f{\cdot}\mathrm{Re}$ vs. Re as shown schematically in Figure 1. The solution of the equations of motion (Bird et al., 1960) for rectilinear laminar flow in the annulus can be represented in the form $f\mathrm{Re} = \phi(\sigma)$ where $\sigma = R_1/R_2$ is the aspect ratio of the annulus. At a value of $\mathrm{Re} = \mathrm{Re}_{ic}$, computed from the theory of Hanks (1963, 1980), the flow changes to some form other than rectilinear laminar motion, characterized by an increase in the product $f{\cdot}\mathrm{Re}$. At some higher value of $\mathrm{Re} = \mathrm{Re}_{oc}$, complete transition to turbulence occurs giving rise to the characteristic steeply rising portion of the curve shown in Figure 1.

Following the suggestion of Rothfus (1948), Hanks and Bonner (1971) modeled this transition region as follows. At $\mathrm{Re} = \mathrm{Re}_{ic}$ the region of flow lying between the core, $\xi = \sigma$, and the position of maximum velocity, $\xi = \lambda$ (Figure 2), called the "inner" or core region, was presumed to undergo a transition to turbulence. The region of flow lying between $\xi = \lambda$ and $\xi = 1$, called the "outer" or wall region, was simultaneously presumed to remain in laminar flow. The inner region flow was then modeled in terms of a transitional flow theory (Hanks, 1968) which had previously been found to be accurate for pipe flow. This dual flow field was presumed to persist until $\mathrm{Re} = \mathrm{Re}_{oc}$, computed by applying Hanks (1963) transition parameter theory to the combined transitional inner-laminar outer flow field. For $\mathrm{Re} > \mathrm{Re}_{oc}$ the entire flow field was modeled in terms of the transitional flow theory (Hanks, 1968). The

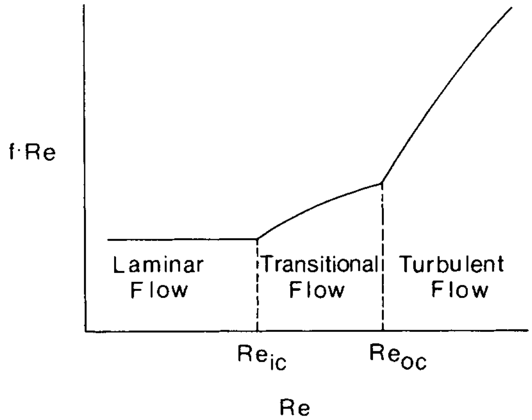

Figure 1. Special transitional flow range in concentric annulus.

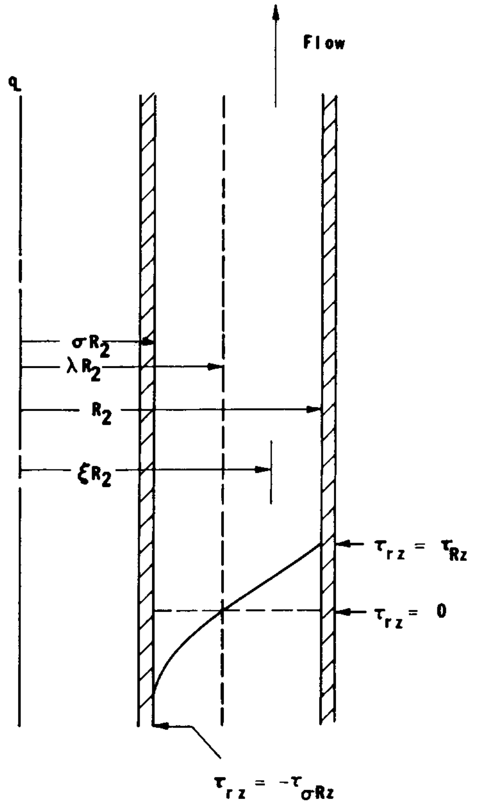

Figure 2. Nomenclature of locations within concentric annuli.

results of this theoretical modeling agreed qualitatively but not quantitatively with their experimental data as illustrated in Figure 3 for an annulus with $\sigma = 0.0455$ where the solid curve was computed from their theory.

Because of the lack of quantitative agreement between the data of Hanks and Bonner (1971) and the theory as illustrated in Figure 3, the experimental study reported here was begun. The objective of the study was to determine whether or not the dual flow model of Hanks and Bonner (1971) was valid.

EXPERIMENTAL

Apparatus

A recirculating flow loop, shown schematically in Figure 4, was constructed and used for all of the experiments described herein. Complete details are available elsewhere (Peterson, 1979).

The working fluids used were aqueous solutions of Polyglycol 15-200.

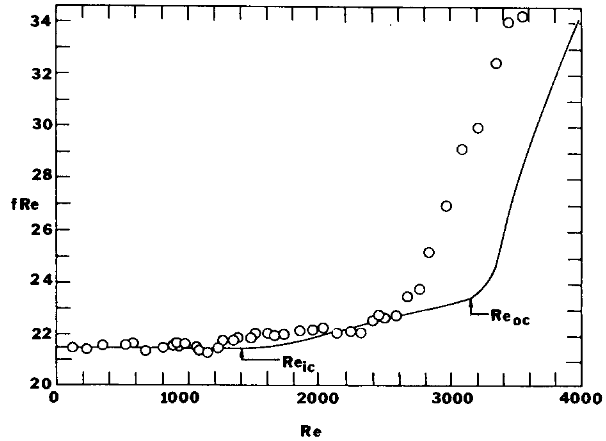

Figure 3. Data of Hanks and Bonner (1971) for annulus (σ = 0.0455) similar to the one used here.

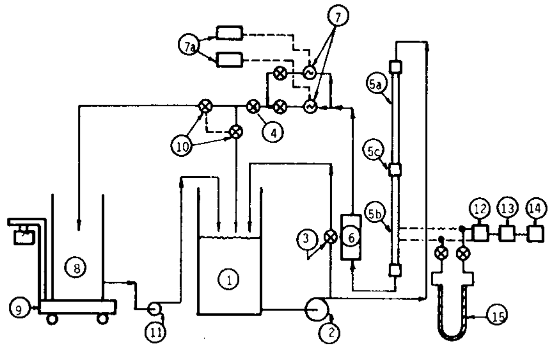

Figure 4. Flow diagram of experimental apparatus: (1) main supply tank, (2) main pump, (3) bypass line and valve, (4) main control valve, (5a) calming section, (5b) test section, (5c) precision pipe connector, (6) filter, (7) turbine flow meters, (7a) counters, (8) auxilliary calibration tank, (9) platform scales, (10) ganged pneumatic valves, (11) calibration tank return pump, (12) pressure transducer, (13) carrier-demodulator, (14) digital voltmeter, (15) calibration manometer.

The data reported here were obtained with a solution having a viscosity of about 0.025 Pa·s (25 cp) and a density of about 1,050 kg m^{-3} at 298 K. By correcting these values for minor temperature variations friction factor data accurate to ±1% were obtained. Temperatures in the flow loop were controlled to 298 ± 0.1 K.

Fluid was pumped from the main supply tank (1) (Figure 4), which was temperature controlled, through the test section (5b), by means of a 18.7 kW (25 hp) centrifugal pump. From there it passes through a micropore filter (6), a turbine flow meter manifold (7) and returns to the supply tank. In-line diversion capability (10) to a dynamic weighing facility (8, 9), used to calibrate the flow meters, allowed spot checking of the turbine flow meter calibrations at any time. All pressure drops were measured by means of a pressure transducer (12)-carrier demodulator (13)-digital voltmeter (14) system. A calibrating manometer (15) was maintained in-line at all times to permit random spot checking of the pressure transducer calibration.

Turbulence intensities were measured using a DISA constant temperature anemometer with a TSI hot film wedge probe. The probe output signal was displayed on a storage oscilloscope from which photographs could be taken as desired.

Results

Figure 5 contains the frictional pressure loss data obtained for this annulus. They are presented in the form f·Re vs. Re to conform to Hanks and Bonner's (1971) method of presentation. Figure 6 shows similar data obtained with the core removed and only the 38.2 mm (1.5 in.) outer pipe being used. These data were obtained as a calibration of the system to prove the integrity of the measurement devices.

In order to assure that the 3.2% rise in the fRe product in Figure 5 was not due to entry length excess pressure drops the data in Figures 7 and 8 were obtained. Figure 7 contains excess pressure gradient data, $P^* = \Delta p_{ij}/\Delta p_\infty$, as a function of Re and the L/D location of the axial pressure tap, where Δp_{ij} is the Δp measured between taps i and j. All pressure taps were spaced at equal length increments along the pipe. Figure 8 contains similar data obtained for the annulus with the 1.588 mm core in place. The data of Figure 7 were analyzed for values of $(L/D\mathrm{Re})_{FD}$ = Z and are compared with several theoretical estimates in Figure 9. The value for Z corresponding to Re = 2165, which is essentially at the transition point exceeded the length of the test section and is listed as "infinity" to indicate that it is some large and undetermined number.

Radial traverses were made with the hot film anemometer at various Reynold numbers to determine whether any measurable turbulence existed. Also, anemometer measurements were made at fixed radial locations for a series of flow rates. Figure 10 shows photographs of oscilloscope traces of anemometer output signals for five different Re values with the probe located at a position indicated by Hanks and Bonner (1971) as the least stable position in the inner or core regions of the annulus. The traces are shown correlated with the fRe vs. Re data of Figure 5. Figure 11 shows similar results but with the probe located at the position designated as the least stable (Hanks and Bonner, 1971) in the outer region of the annulus.

DISCUSSION OF RESULTS

The data presented in Figure 6 show clearly that the pipe which forms the outer boundary of the annulus responds exactly according

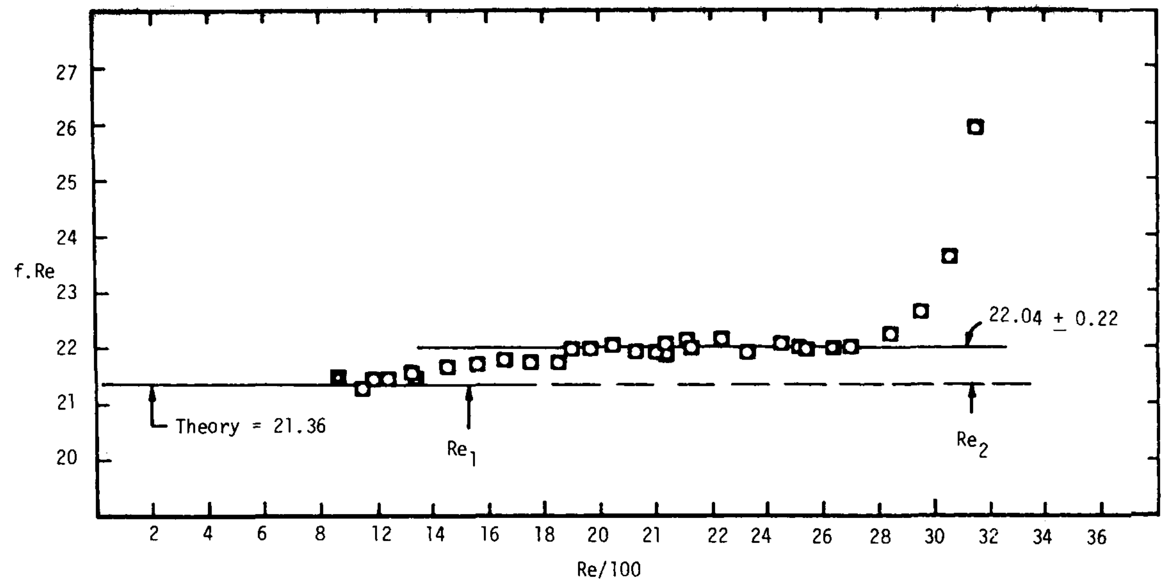

Figure 5. f·Re vs. Re data for flow in a concentric annulu (σ = 0.0416).

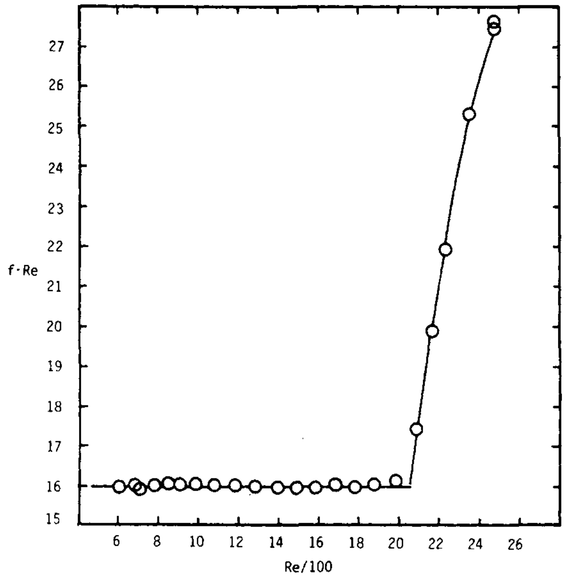

Figure 6. Pipe flow calibration data using outer pipe only.

to laminar pipe-flow theory. The fluid in this pipe flows in rectilinear laminar motion up to the transition point at Re $\simeq$ 2,050. At this point a normal transition to turbulence appears to occur.

The data presented in Figure 5 were obtained after installation of a 1.588 mm steel wire along the axis of the pipe. Great care was exercised (Peterson, 1979) to ensure that this wire was precisely concentric with the outer pipe. The point labeled Re_1, in Figure 5 corresponds to the first critical Reynolds number of transition as predicted by Hanks' theory (Hanks and Bonner, 1971; Hanks, 1980). For Re < Re_1, the data cluster about fRe = 21.45 ± 0.24 (95% confidence level) which includes the theoretical value 21.36, thus confirming that for Re < Re_1, rectilinear laminar flow occurred.

The second critical Reynolds number predicted by Hanks and Bonner (1971) is labeled Re_2 in Figure 5. As was the case with their data this value is seen to be too large when compared with the experimental data. For flows in the range Re_1 < Re < 2,800 the fRe data have an average value of 22.04 ± 0.22 (95% confidence level). This value excludes the theoretical laminar value and the difference is statistically significant. The apparatus used by Bonner (Hanks and Bonner, 1971) was completely different from the present system. Therefore, we conclude that the 3.2% increase in fRe is real and reproducible.

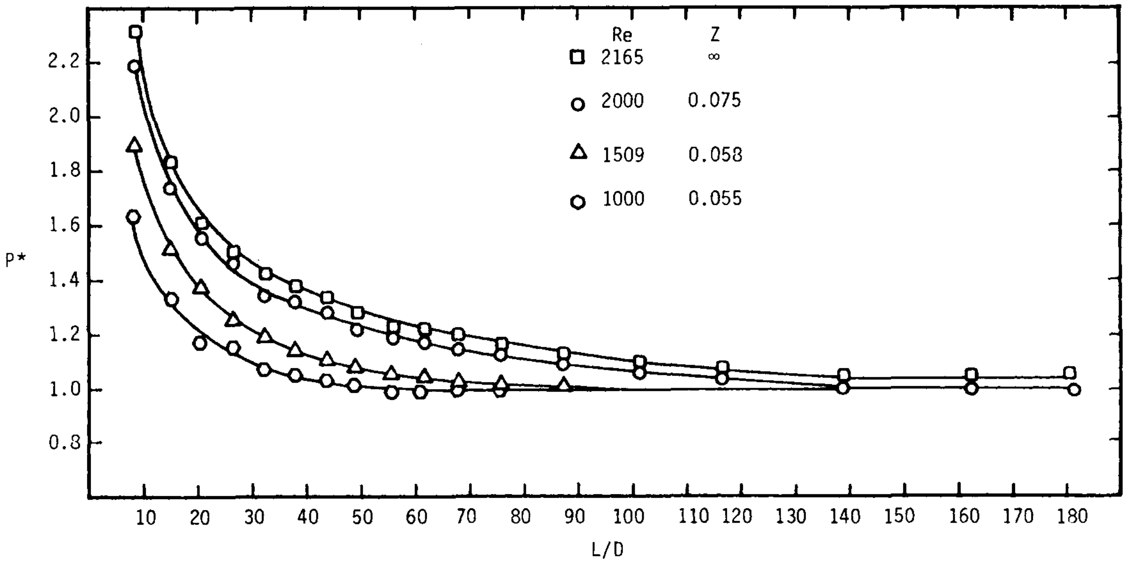

Figure 7. Axial pressure drop profiles for 1.5 in I.D. outer pipe. Value $Z = \infty$ for Re = 2,165 indicates Z exceeded length of experimental pipe and was a large undetermined value.

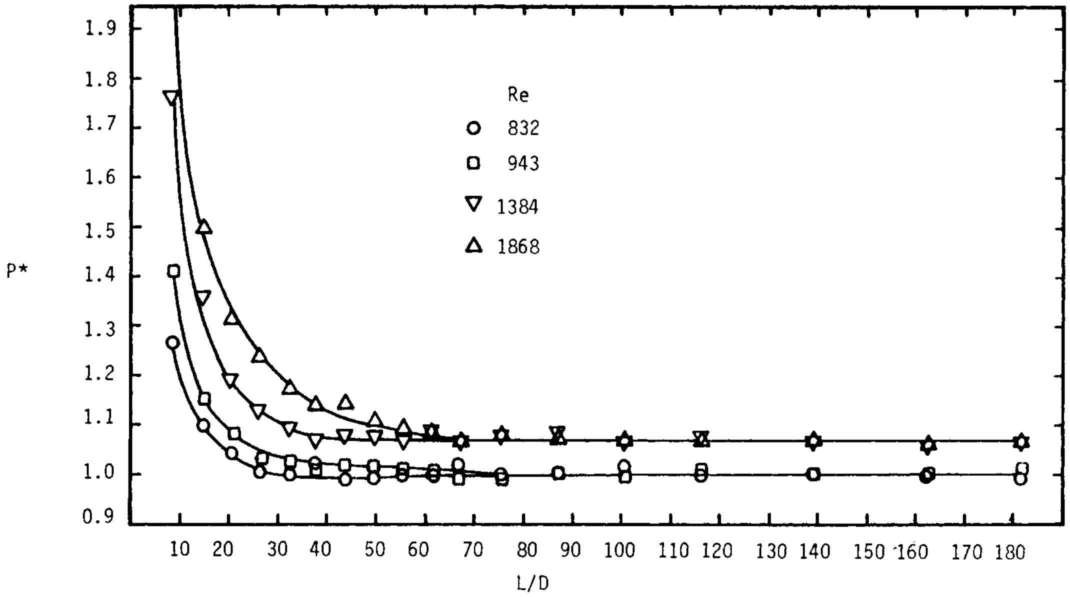

Figure 8. Axial pressure drop profiles for annulus (σ = 0.0416) at four different values of Re.

One possible explanation for this increase in fRe which had to be considered is that it might be due to an insufficient entrance length so that the flow is still developing and exhibits an excess pressure gradient. In order to examine this possibility pressure taps were installed at several uniformly spaced points along the pipe and the data shown in Figure 7 were obtained. From these results it is evident that no entrance length problems are encountered in the open pipe. Figure 9 is another way of comparing these data with theory to verify the entrance length integrity of the pipe. The pipe is seen to be sufficiently long that all data obtained in the last downstream pressure taps are exactly in agreement with laminar flow theory. Therefore, the open pipe appears to be free of any spurious effects of excess pressure drops which might confuse the issue.

Figure 8 contains a set of axial pressure profiles obtained for the annulus with the core in place. These data span a Re range which includes the first transition at Re_1 = 1,320 shown in Figure 5. A very distinct difference is observed between the data in Figure 8 and those in Figure 7 for the open pipe. It is obvious from Figure 8 that for Re = 832 and 943, both of which are less than Re_1 = 1,320, the flow is fully developed over a large portion of the annulus. For Re = 1384 and 1868, both of which are greater than Re_1 = 1320, and clearly lie within the region where the statistically significant increase in fRe occurs, a very interesting result is observed. The pressure profiles obtained clearly show the behavior of a fully developed flow just as in the rectilinear laminar flow case, with the exception that the fully developed axial pressure gradient lies at a higher level than that for rectilinear laminar flow. Clearly, therefore, the flow field which causes the increased pressure gradient in the range $Re_1 < 2,800$ is a *fully developed flow* which is *not* rectilinear laminar flow.

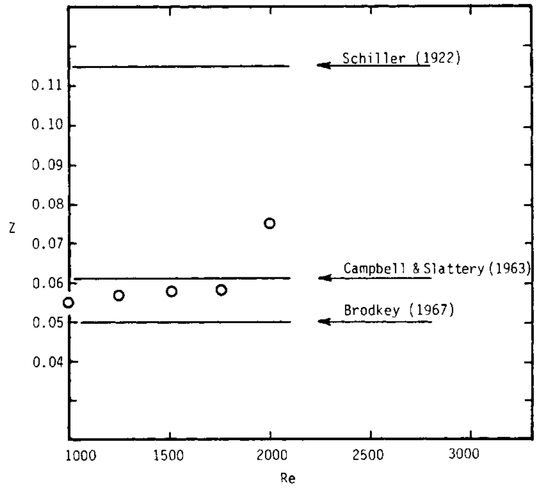

Figure 9. Fully developed length coefficient $Z = (L/DRe)_{FD}$ for external pipe in comparison with various theories.

Hanks and Bonner (1971) developed a theory to explain this flow. Their theory was based on the assumption that near the core of the annulus a macroscopic zone of turbulence developed which was surrounded by a concentric macroscopic zone of laminar flow. This theory gave rise to theoretical fRe vs. Re curves having qualitatively the shapes observed, but not in quantitative agreement as illustrated in Figure 3 where the solid curve is calculated from their theory. In order to determine whether this theoretical model was valid a series of measurements was made with a hot film anemometer probe. In the theory developed by Hanks and Bonner (1971) the values of Re_1 and Re_2 were calculated by application of Hanks' (1963) transition parameter theory to the two regions of flow separated by the radius of maximum velocity. This operation identifies two locations, one in each zone, where the transition parameter, K, has its maximum value (Hanks, 1980) corresponding to a condition of minimum stability. Supposedly at these two locations disturbances should most readily grow into turbulence. These two radial positions are designated as the $K_{\max\ in}$ and $K_{\max\ out}$ locations in Figures 10 and 11, respectively.

A series of tests were run in which the hot film anemometer was positioned at one or the other of these two locations. The flow rate was then increased in small steps from low to high values so as to cover the entire range of Reynolds numbers spanned by the fRe data in Figure 5. At each flow rate step the output signal from the hot film anemometer was displayed on an oscilloscope screen and photographically recorded. This technique was adopted after numerous radial traverses made at various flow rates showed that no additional information could be gained from the more difficult method of traversing. Selected ones of the photographs obtained in this manner, which are typical of the intermediate Reynolds number ranges, are shown in Figures 10 and 11 correlated with the particular Re value in the fRe vs Re data of Fig. 5 to which each corresponds. Figure 10 corresponds to the probe being positioned at the point of minimum stability in the inner region between the core and the position of maximum velocity, and Figure 11 corresponds to the probe being positioned at the same point in the outer region between the position of maximum velocity and the outer wall.

From the pictures of the anemometer traces in Figures 10 and

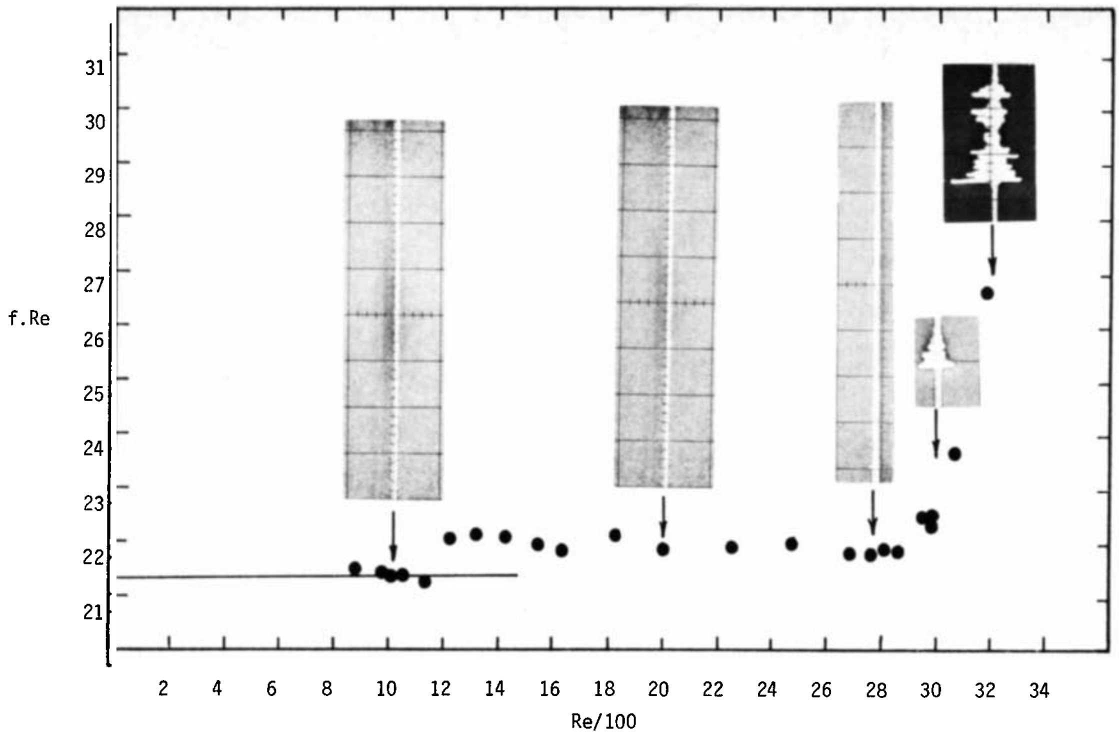

Figure 10. f·Re vs. Re for flow in an annulus (σ = 0.0416) with oscilloscope traces of hot film anemometer at various Re values. Probe is located at $K_{max\ in}$ location. See text for explanation.

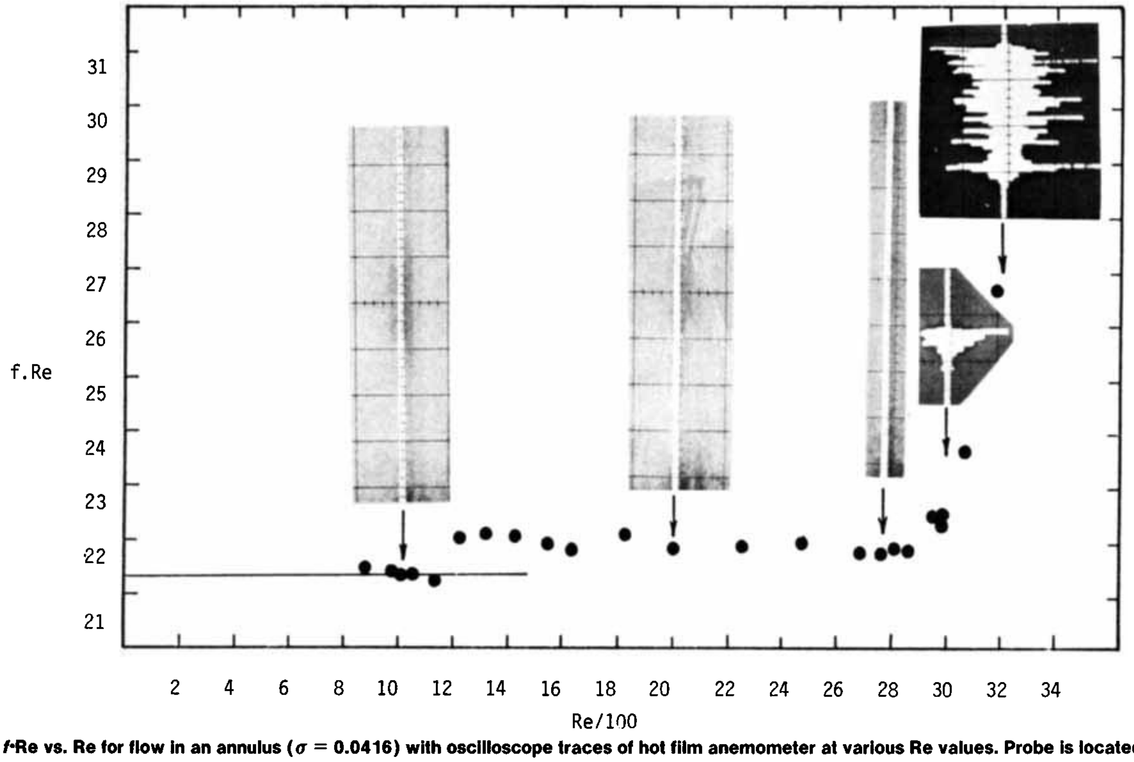

Figure 11. f·Re vs. Re for flow in an annulus (σ = 0.0416) with oscilloscope traces of hot film anemometer at various Re values. Probe is located at $K_{max\ out}$ location. See text for explanation.

11 it is immediately clear that no measurable turbulence exists in the Re range $Re_1 < Re < 2{,}800$. The oscilloscope traces in this range could be observed for extended time periods with no evidence of any turbulence, even on a random or intermittent basis. Above Re = 2,800 the oscilloscope traces show an increasing frequency of turbulent bursts with increasing Re exactly as was observed in the open pipe around Re = 2,000. This clearly indicates that around Re = 2900, the location of the very steeply rising break in the fRe curve in Figure 5, a transition to turbulence similar in character to the type normally observed in a pipe occurs. However, at Re = Re_1 the transition from rectilinear laminar flow which occurs results in some other type of nonturbulent but non-rectilinear laminar flow. Clearly, therefore, the model assumed by Hanks and Bonner (1971) and proposed by Rothfus (1948) and Lea and Tadros (1931) much earlier, is incorrect. There is no macroscopic turbulent region near the core surrounded by a macroscopic annular laminar zone. It is, therefore, not at all surprising that Hanks and Bonner's (1971) theory did not agree quantitatively with their data.

CONCLUSIONS

The data presented here have demonstrated conclusively the following points:

- Two critical Reynolds numbers exist for axial flow in a concentric annulus.
- Each corresponds to a transition in flow type.
- Below the first critical Reynolds number the flow which occurs is rectilinear laminar flow.
- For a wide range of Reynolds numbers between the two critical values a steady, fully developed type of flow exists for which the fRe product is larger than that for rectilinear laminar flow by a reproducibly measurable, statistically significant amount. In the present annulus this increase is 3.2%. According to the data of Hanks and Bonner (1971) the magnitude of this increase in f.Re is a function of the aspect ratio $\sigma = R_1/R_2$.

NOTATION

D	= diameter of pipe
D_H	= hydraulic diameter = $2(R_2 - R_1)$
f	= friction factor = $D_H \Delta p / 2L\rho \langle v \rangle^2$
K	= transition parameter
L	= length of duct
r	= radial position
P^*	= excess axial pressure gradient = $\Delta p_{ij}/\Delta p_\infty$
Δp	= pressure drop
Δp_{ij}	= pressure drop between taps i and j
Δp_∞	= fully developed pressure drop
R_1	= core radius
R_2	= pipe radius
Re	= Reynolds number = $D_H \langle v \rangle \rho/\mu$
$\mathrm{Re}_{ic}, \mathrm{Re}_{oc}$	= critical Reynolds numbers
$\langle v \rangle$	= average velocity of flow
Z	= $(L/D\mathrm{Re})_{FD}$
λ	= value of ξ at maximum velocity
μ	= fluid viscosity
ξ	= r/R_2
ρ	= fluid density
σ	= annulus aspect ratio = R_1/R_2
ϕ	= general functional notation

LITERATURE CITED

Bird, R. B., W. E. Stewart, and E. N. Lightfoot, "Transport Phenomena," J. Wiley, New York (1960).

Brodkey, R. W., "The Phenomena of Fluid Motions," Addison-Wesley, Reading, MA (1967).

Campbell, W. D., and J. C. Slattery, "Flow in the Entrance of a Tube," *J. Basic Engr.*, **85**, 41 (1963).

Croop, E. J., "Velocity Distribution in Transitional Flow Through Annuli," Ph.D. Dissertation, Carnegie Tech. (1958).

Croop, E. J., and Rothfus, R. R., "Skin Friction Patterns for Transitional Flow in Annuli," *AIChE J.*, **8**, 26 (1962).

Hanks, R. W., "Critical Reynolds Number for Newtonian Flow in Concentric Annuli," *AIChE J.*, **26**, (1), 152 (1980).

Hanks, R. W., "On the Theoretical Calculation of Friction Factors for Laminar, Transitional and Turbulent Flow of Newtonian Fluids in Pipes and Between Parallel Plane Walls," *AIChE J.*, **14**, 691 (1968).

Hanks, R. W., "The Laminar-Turbulent Transition for Flow in Pipes, Concentric Annuli, and Parallel Plates," *AIChE J.*, **9**, 45 (1963).

Hanks, R. W., and W. F. Bonner, "Transitional Flow Phenomena in Concentric Annuli," *Ind. Eng. Chem. Fund.*, **10**, 105 (1971).

Joseph, D. D., "Stability of Fluid Motions I," Springer-Verlag, Berlin, Chap. 4 (1976).

Lea, F. C., and A. G. Tadros, "Flow of Water Through a Circular Tube with a Central Core and Through Rectangular Ducts," *Phil. Mag.*, **11**, 1235 (1931).

Mildenahll, G. T., "Linear Stability Analysis in Annuli by Chebyshev Expansion," Ph.D. Dissertation, Brigham Young University (1979).

Mott, J. E., and D. D. Joseph, "Stability of Parallel Flows Between Concentric Cylinders," *Phys. Fluids*, **11**, 2065 (1968).

Narvaez, C., "The Temporal Stability of Laminar Flows in Concentric Annuli," Ph.D. Dissertation, Brigham Young University (1977).

Peterson, J. M., "Transitional Flow in an Annulus," Ph.D. Dissertation, Brigham Young University (1979).

Peterson, J. M., and R. W. Hanks, "Transitional Velocity Profiles in a Concentric Annulus," *I & EC Fund.*, 1981 (submitted).

Rothfus, R. R., "Velocity Distribution and Fluid Friction in Concentric Annuli," Ph.D. Dissertation, Carnegie Tech. (1948).

Rothfus, R. R., C. C. Monrad, and V. E. Senecal, "Velocity Distribution and Fluid Friction in Smooth Concentric Annuli," *Ind. Eng. Chem.*, **42**(12), 2511 (1950).

Rothfus, R. R., J. E. Walker, and G. A. Whan, "Correlation of Local Velocities in Tubes, Annuli, And Parallel Plates," *AIChE J.*, **4**, 240 (1958).

Rotta, J., "Experimentaller Beitrag zur Entstehung turbulenter Stromung im Rohr," *Ing.-Archiv.*, **24**, 258 (1956).

Schiller, L., "Untersuchungen uber laminare and turbulente Stromung," *Phys. Z.*, **23**, 14 (1922).

Walker, J. E., Whan, G. A., and Rothfus, R. R., "Fluid Friction in Noncircular Ducts," *AIChE J.*, **3**, 484 (1957).

Wygnanski, I. J., and F. H. Champagne, "On the Transition in a Pipe. Part 1. The Origin of Puffs and Slugs and the Flow in a Turbulent Slug," *J. Fluid Mech.*, **59**(2), 281 (1973).

Manuscript received June 15, 1979; revision received November 23, 1981 and accepted January 13, 1982.